In order to obtain a consistent set of equations (B), the following sign convention should be observed.

$\frac{\partial\Psi}{\partial n}$ (which we have seen has the approximate value $\frac{(\Psi_1-\Psi_2)}{t}$) is positive if Ψ increases as we move outwards along a normal to the boundary considered.

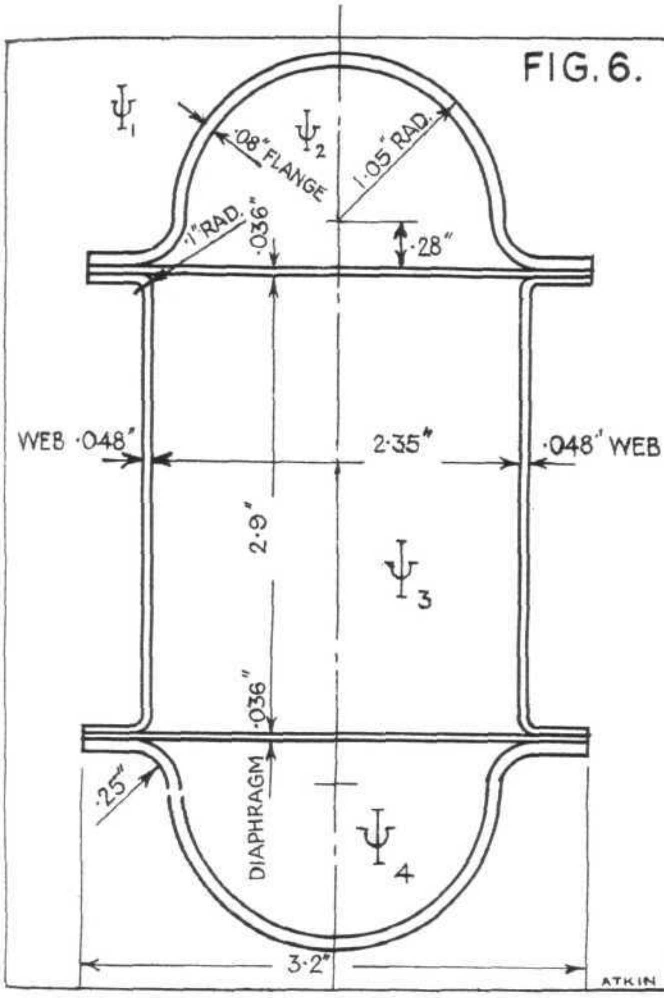

FIG. 6.

To make the method of forming the equations quite clear, let us take as an example the spar section in Fig. 6. The various areas and lengths are indicated on the diagram.

By symmetry $\Psi_2 = \Psi_4$.

Hence, there are two equations (B),

$$\frac{3\cdot888}{0\cdot08}(\Psi_1-\Psi_2)+(\Psi_3-\Psi_2)\frac{2\cdot6}{0\cdot036}+4\cdot134=0\ldots \text{(i)}$$

$$\frac{2\times3\cdot014}{0\cdot048}(\Psi_1-\Psi_3)+\frac{2\times2\cdot55}{0\cdot036}(\Psi_2-\Psi_3)+13\cdot66=0\ldots \text{(ii)}$$

or, putting $\Psi_1 = 0$, and simplifying,

$$-120\cdot9\,\Psi_2+72\cdot3\,\Psi_3+4\cdot134=0$$
$$141\cdot7\,\Psi_2-267\cdot4\,\Psi_3+13\cdot66=0$$

and finally

$$\Psi_2 = 0\cdot0946$$
$$\Psi_3 = 0\cdot1012.$$

Equation (A) for the torque becomes

$$T = G\theta[2\Psi_3A_3+4\Psi_2A_2-2\Psi_1A_1 + \Psi_1A_{\Psi1}+2\Psi_2A_{\Psi2}+\Psi_3A_{\Psi3}]$$

so that θ can now be found in terms of T_1 and G.

Hence

$$\theta = \frac{T}{2\cdot29\,G}\text{ Rads. per in.}$$

and stress in flanges $= G\theta\frac{(\Psi_1-\Psi_2)}{t_{\text{flange}}} = 0\cdot516\text{T lb./in.}^2$

also, stress in webs $= G\theta\frac{(\Psi_1-\Psi_3)}{t_{\text{web}}} = 0\cdot921\text{T lb./in.}^2$

and, stress in diaphragms $= G\theta\frac{(\Psi_2-\Psi_3)}{t_{\text{diaphragm}}} = 0\cdot081\text{T lb./in.}^2$

T being the applied torque in lb. in.^{2}

In many practical cases the section is symmetrical about one or more axes: it may then be possible to see by inspection, the relative values of the shears in some parts of the section, and to obtain an idea of the efficiency of a given type of section in torsion. Take, for instance, the series of sections in Fig. 7.

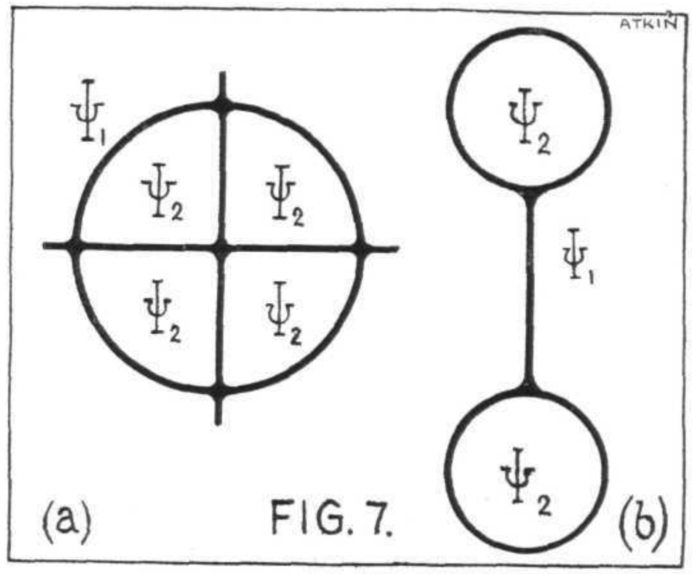

FIG. 7.

Section (a) is seen to have equal values of Ψ inside each of the inner boundaries so that there is no stress in the cruciform portion of the section. As the stiffness of a very narrow rectangle is negligible, this part of the section is, therefore, redundant.

Again, the single web spar (b) is seen to be an inefficient section in torsion because Ψ has the same value on both sides of the web, which is, therefore, ineffective.

This weakness of the single web spar is well known, and may be contrasted with the double-web spar of Fig. 6, which is much more efficient torsionally.

It was mentioned at the beginning that the value of the shear stress obtained by the method here described is only true when the thickness at any point of the section is small compared with the radius of curvature.

It is now proposed to outline the method of determining the maximum shear stress in a corner where the radius to thickness ratio is small.

Having analysed the section, and determined the mean shear stress, the following formula[6] may be used for single bends (no formula appears to exist for corners such as A, B, C, or D in Fig. 5).

$$f_{s\,\text{max.}} = \frac{f_{s\,\text{mean}}\,1}{r_i}\cdot\frac{\left[1-\frac{G\theta}{2f_{s\,\text{mean}}}(r_o+r_i)\right]}{\log_e\frac{r_o}{r_i}} + r_iG\theta$$

Where r_o = outside radius of bend.
and r_i = inside radius of bend.

Also, as before, t = thickness of section wall.

In the case of the 0·1 in. radius in the web of Fig. 5, the maximum shear stress is 28 per cent. greater than the mean. This maximum occurs on the concave side of the bend.

In a sharp corner the stress is theoretically infinite, but this does not necessarily indicate failure. The importance of the concentration of stress in a corner of large or infinite curvature depends on the material of which the section is made. In the case of a ductile material such as mild steel the theoretical results may be of little value; but in the case of a very brittle material, a high theoretical value for the shear stress concentration is a true indication that failure will occur.

(6) *Strength of Materials*, Vol. 2, Timoschenko.